MATH 16A MIDTERM 1(PRACTICE 2)
PROFESSOR PAULIN

DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.

CALCULATORS ARE NOT PERMITTED

YOU MAY USE YOUR OWN BLANK PAPER FOR ROUGH WORK

SO AS NOT TO DISTURB OTHER STUDENTS, EVERYONE MUST STAY UNTIL THE EXAM IS COMPLETE

REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT

THIS EXAM WILL BE ELECTRONICALLY SCANNED. MAKE SURE YOU WRITE ALL SOLUTIONS IN THE SPACES PROVIDED. YOU MAY WRITE SOLUTIONS ON THE BLANK PAGE AT THE BACK BUT BE SURE TO CLEARLY LABEL THEM

Name and section: ______________________________

GSI's name: ______________________________

This exam consists of 5 questions. Answer the questions in the spaces provided.

1. (25 points) (a) Determine the equation of the straight (in the xy-plane) which contains the points $(1, 4)$ and $(-1, 3)$

Solution:

$$y - 4 = \frac{3-4}{-1-1}(x-1)$$

$$\Rightarrow \quad y - 4 = \frac{1}{2}(x-1)$$

(b) Calculate the x-intercept of the straight line containing $(3, 2)$, which is perpendicular to the line in (a).

Solution:

Perpendicular slope $= \dfrac{-1}{\left(\frac{1}{2}\right)} = -2$

$$\Rightarrow \quad y - 2 = -2(x-3)$$

$$\Rightarrow \quad y = -2x + 8$$

$y = 0 \Rightarrow -2x + 8 = 0 \Rightarrow x = 4$

$\Rightarrow$ x-intercept is at 4

2. (25 points) A product is to be produced and sold. The cost function $C(x)$ is linear, with marginal cost of 1 and fixed cost of 4. The revenue function is

$$R(x) = x^2 + x$$

(a) Determine the break-even quantity.

Solution:

marginal cost

fixed cost

$$C(x) = 1 \cdot x + 4 = x + 4$$

$$C(x) = R(x) \Rightarrow x + 4 = x^2 + x \Rightarrow x^2 = 4$$

$$\Rightarrow x = 2 \quad (x = -2 \text{ has no meaning})$$

Break even quantity

(b) What is the marginal profit at this quantity? You must calculate this from first principles using limits.

Solution:

$$P(x) = R(x) - C(x) = x^2 - 4$$

$$\text{Marginal Profit} = P'(x) = \lim_{h \to 0} \frac{P(x+h) - P(x)}{h}$$

$$= \lim_{h \to 0} \frac{((x+h)^2 - 4) - (x^2 - 4)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{-4} - \cancel{x^2} \cancel{+4}}{h}$$

$$= \lim_{h \to 0} 2x + h = 2x$$

3. Calculate the following limits. If they do not exist determine if they are ∞ or $-\infty$.

(a)

$$\lim_{x\to -1} \frac{x^2+3x+2}{x^2-x-2}$$

Solution:

$$\frac{x^2+3x+2}{x^2-x-2} = \frac{(x+1)(x+2)}{(x-2)(x+1)} = \frac{x+2}{x-2}$$

$$\Rightarrow \lim_{x\to -1} \frac{x^2+3x+2}{x^2-x-2} = \lim_{x\to -1} \frac{x+2}{x-2} = \frac{-1+2}{-1-2} = \frac{1}{-3}$$

(b)

$$\lim_{x\to -\infty} \sqrt[3]{\frac{4x}{7x^2+5}}$$

degree 1

degree 2

Solution:

$$\lim_{x\to -\infty} \frac{4x}{7x^2+5} = 0 \Rightarrow \lim_{x\to -\infty} \sqrt[3]{\frac{4x}{7x^2+5}} = \sqrt[3]{0} = 0$$

(c)

$$\lim_{x\to 1^-} \frac{1-x}{x^2-2x+1}$$

Solution:

$$\frac{1-x}{x^2-2x+1} = \frac{1-x}{(1-x)^2} = \frac{1}{1-x}$$

$$\lim_{x\to 1^-} 1 = 1 > 0$$

$$\lim_{x\to 1^-} 1-x = 0^+$$

1

$1-x>0$ $\quad$ $1-x<0$

$$\Rightarrow \lim_{x\to 1^-} \frac{1-x}{x^2-2x+1} = \lim_{x\to 1^-} \frac{1}{1-x} = \infty$$

(DNE)

4. Let $f(x) = \begin{cases} \frac{x-1}{\sqrt{x}-1} + a^2 & \text{if } x > 1 \\ a - x^2 & \text{if } x \le 1 \end{cases}$ for some real number a.

Is it possible to choose a real number a such that $f(x)$ continuous at $x = 1$? Carefully justify your answer.

Solution:

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \left(\frac{x-1}{\sqrt{x}-1} + a^2 \right)$$

$$= \lim_{x \to 1^+} \left(\frac{(\sqrt{x}+1)(\sqrt{x}-1)}{\sqrt{x}-1} + a^2 \right)$$

$$= \lim_{x \to 1^+} \left(\sqrt{x} + 1 + a^2 \right) = \sqrt{1} + 1 + a^2 = 2 + a^2$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} a - x^2 = a - 1^2 = a - 1$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) \Rightarrow 2 + a^2 = a - 1$$

$$\Rightarrow a^2 - a + 3 = 0$$

$$\Rightarrow a = \frac{1 \pm \sqrt{(-1)^2 - 12}}{2}$$

negative so no real solutions

$\Rightarrow$ No value of a will give a continuous function

5. Using limits, calculate the derivative of $f(x) = \frac{x}{x+1}$. Determine the points on the graph $y = f(x)$ where the slope of the tangent line is 2.

Solution:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x+h}{x+h+1} - \frac{x}{x+1}}{h}$$

$$= \lim_{h \to 0} \frac{(x+1)(x+h) - x(x+h+1)}{h(x+h+1)(x+1)}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + \cancel{xh} + \cancel{x} + h - \cancel{x^2} - \cancel{xh} - \cancel{x}}{h(x+h+1)(x+1)}$$

$$= \lim_{h \to 0} \frac{1}{(x+h+1)(x+1)} = \frac{1}{(x+1)^2}$$

$$f'(x) = 2 \Rightarrow \frac{1}{(x+1)^2} = 2 \Rightarrow (x+1)^2 = \frac{1}{2}$$

$$\Rightarrow x = -1 \pm \sqrt{\tfrac{1}{2}}$$

END OF EXAM